powerphotonic
enhancing beam performance

Application Note AN001

Form Error Definition for Freeform Surfaces

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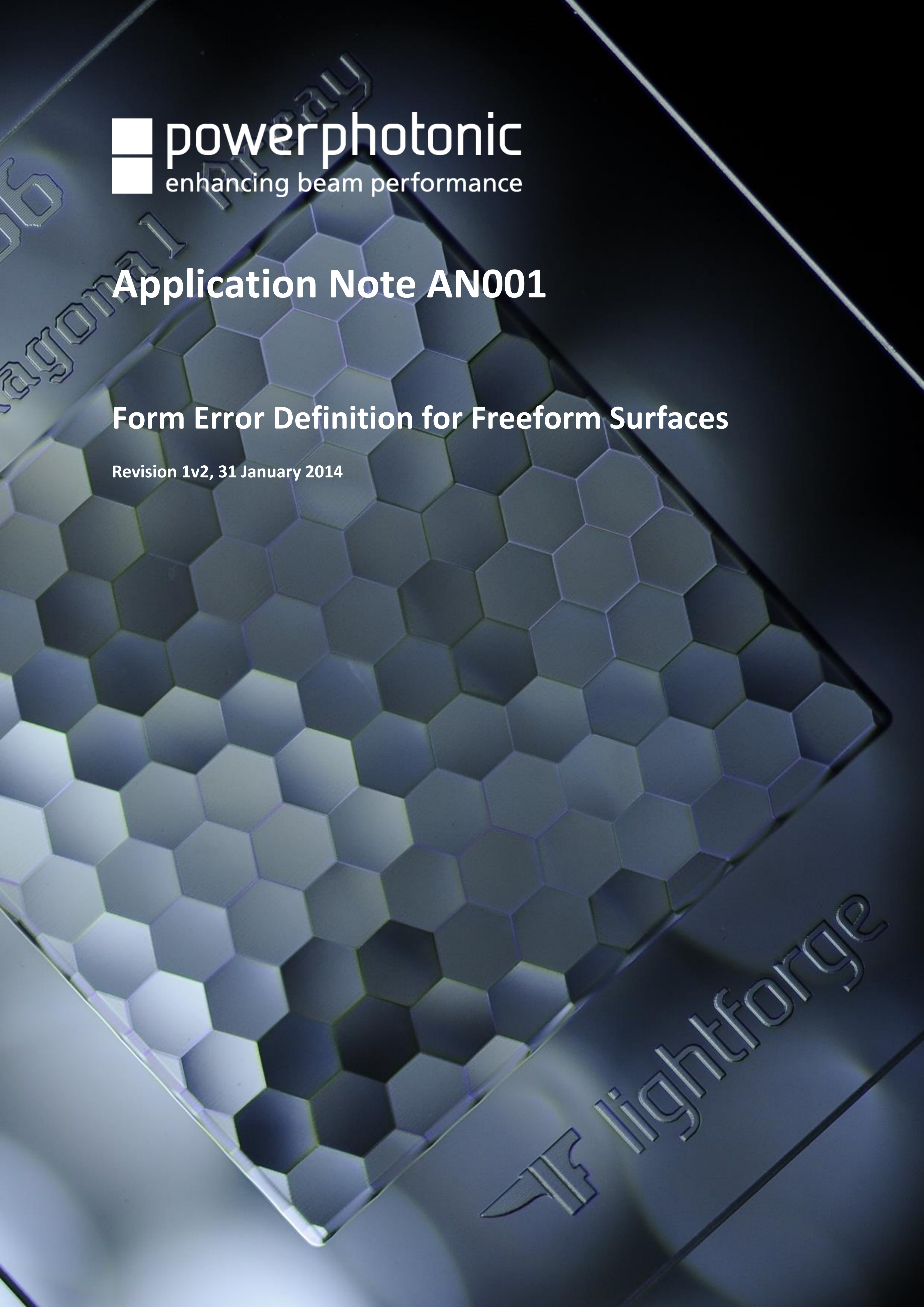


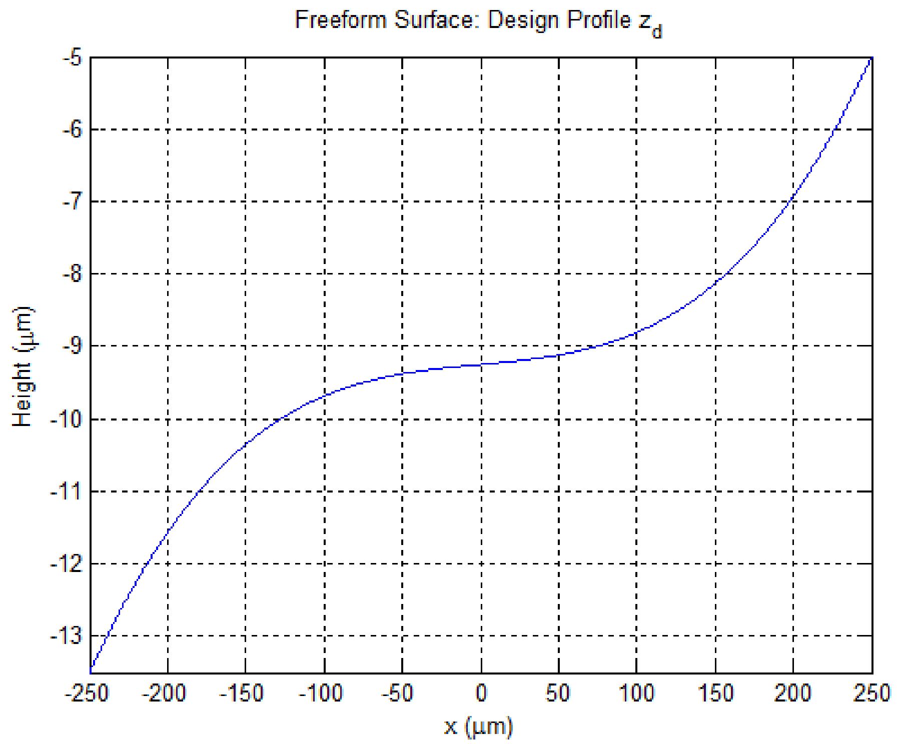
FORM ERROR OVERVIEW

Form error is the term used to describe a quantitative deviation from the ideal surface shape. While ISO 10110 describes how to measure the errors associated with spherical and aspherical surfaces such as lens arrays, the deviation from an ideal freeform optical surface does not yet have a universal definition. As with any manufacturing process, a real surface profile will differ from its ideal design by some degree. This document aims to describe the method used by PowerPhotonic when comparing an ideal design with the resulting manufactured optical surface for freeform surfaces.

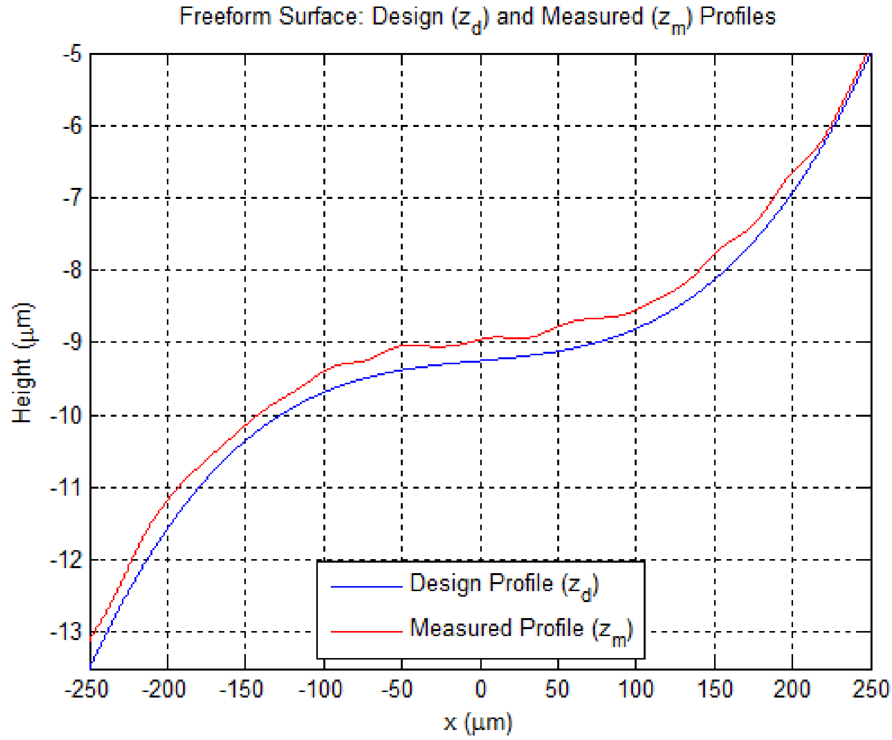
FORM ERROR DEFINITION FOR FREEFORM SURFACES

PowerPhotonic's form error definition for freeform surfaces is applicable where any given local surface slope, as measured from the planar substrate surface, is relatively small (<10°). Generally the surface is a map of height *z* over the *x,y* plane, i.e. $z = z(x, y)$.

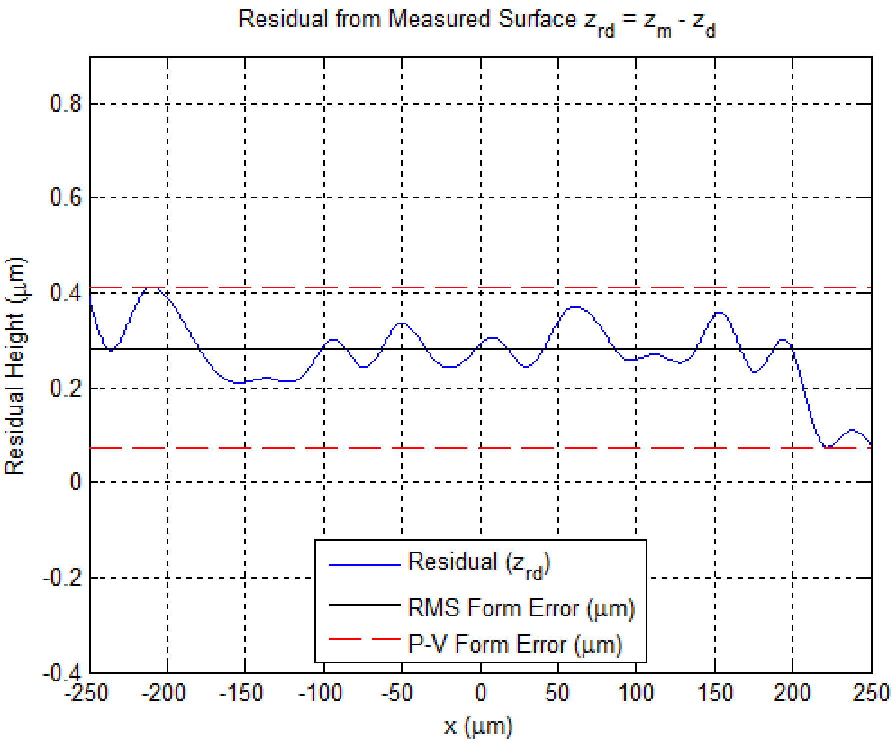
To explain the process of form error calculation, a freeform surface has been designed; random noise and a scaling factor has been applied in order to simulate a real surface for *illustrative purposes only*.



The design profile z_d has been modified with a scaling factor and smoothed random noise in order to simulate a real freeform optical surface (z_m):

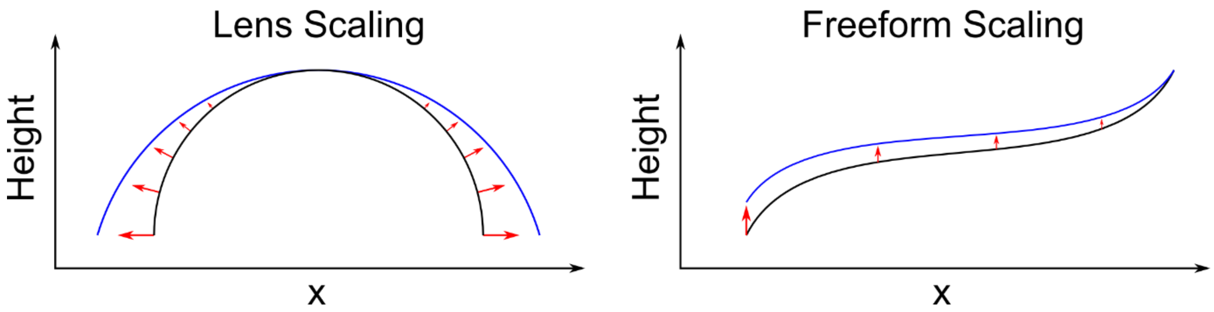


The **measurement residual** z_{rd} is then found by subtracting the design profile z_d from the measured profile z_m:



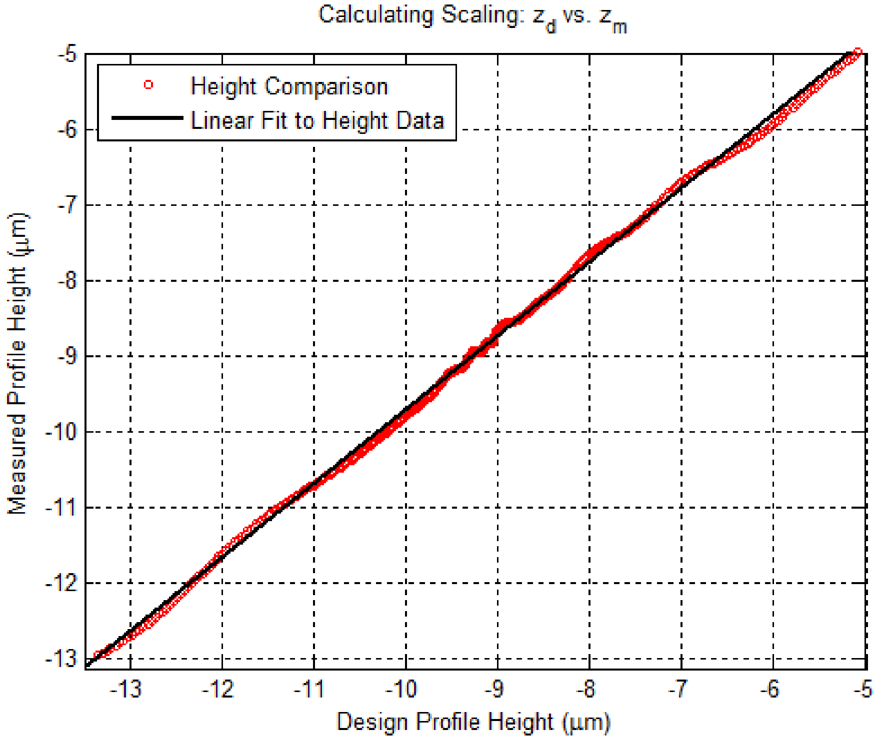
Scaling error is a term used to describe the linear depth error between the design and measured surfaces. This is analogous to the Radius of Curvature (RoC) error that describes traditional rotationally symmetric lenses, which is calculated by scaling the design surface with a vector normal to the lens surface. By converting this RoC error to focal length error the error value that is optically significant is obtained.

Because a truly freeform structure cannot be expected to have any axes of symmetry or focal length, it is nonsensical to describe these surfaces using the same methodology as applied to rotationally symmetric lenses. Instead, freeform surface scaling occurs along vectors parallel to the z or height axis. This allows a scaling error value to be obtained that relates exclusively to the surface rather than the optical impact of this scaling error. Consider the following diagram:

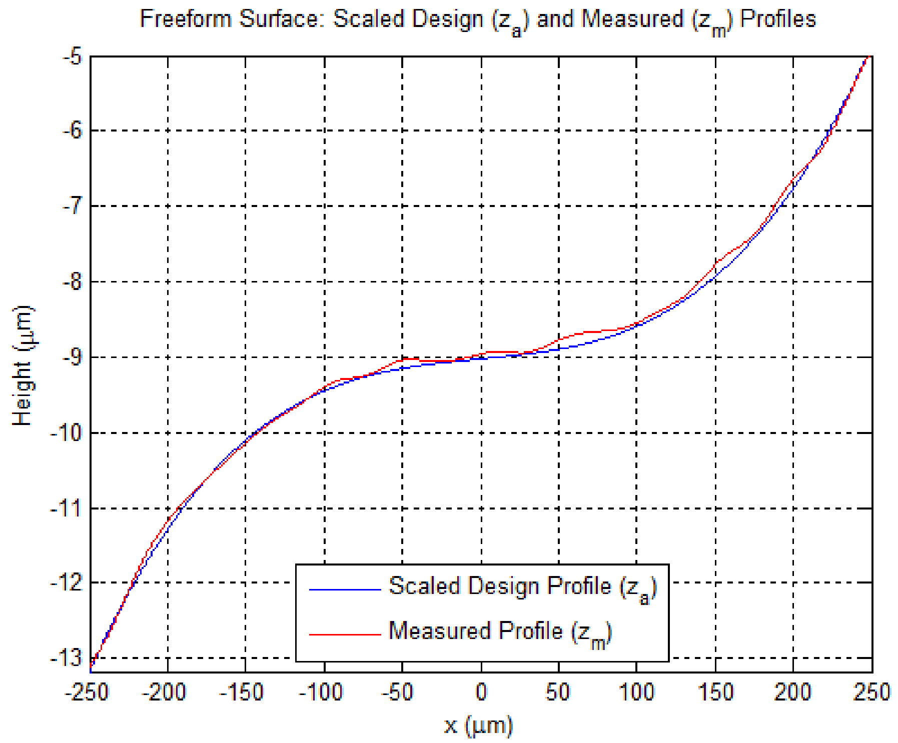


Lens scaling compared to freeform scaling: the blue curve denotes the measured surface, the black curve denotes the design surface, and the red arrows show the direction in which scaling error is calculated.

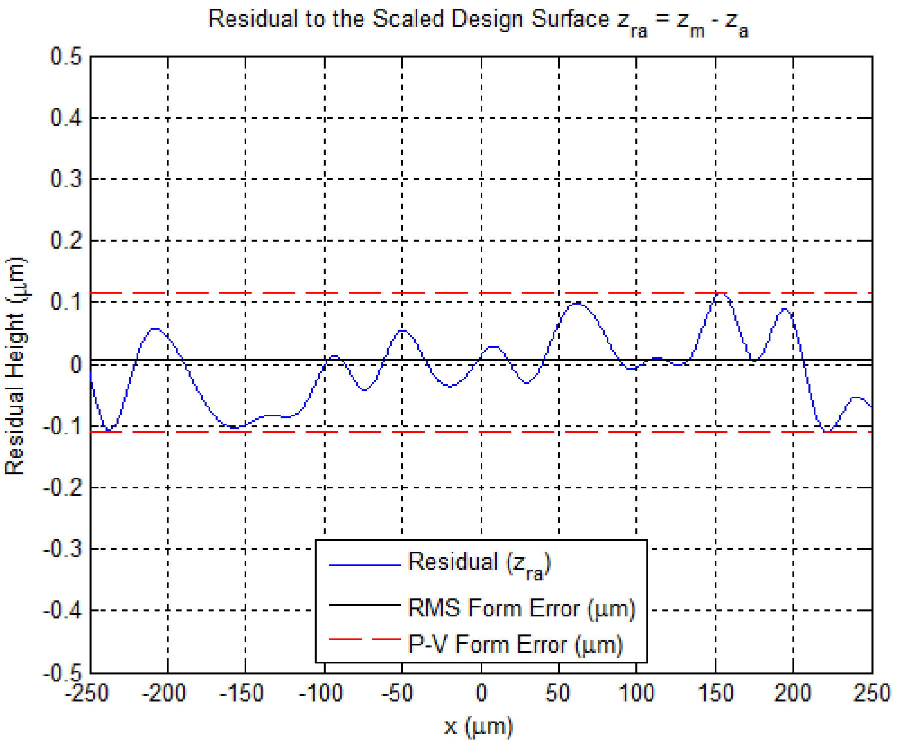
The **scaling error** *C* can be accurately determined by plotting the design profile height z_d against the measured profile height z_m and finding the gradient of a linear fit (m) to these data points:



The **scaled design surface** z_a is then obtained by multiplying the design surface z_d by m:



The **residual to the scaled surface** z_{ra} is then found by subtracting the **scaled design surface** z_a from the measured surface z_m:



So for a freeform structure whose design surface is described by $z_d(x,y)$, and whose measured surface is described by $z_m(x,y)$ we define the following:

- The **measurement residual** $z_{rd}(x,y) = z_m(x,y) - z_d(x,y)$
- The **scaled design surface** $z_a(x,y) = m\, z_d(x,y)$
- The **residual to the scaled surface** $z_{ra}(x,y) = z_m(x,y) - z_a(x,y)$
- The **scaling factor** $m = gradient\ of\ linear\ fit\ to\ data\,\frac{z_m(x,y)}{z_d(x,y)}$ (defined so as to minimise the RMS value of $z_{ra}(x,y)$)
- The **scaling error** $C = m - 1$

Where the optic is fabricated on a plane-parallel substrate and is intended for use in transmission, the effects of wafer bow may be compensated for by subtracting the estimated wafer bow $z_b(x,y)$ from z_m.

In the above, z_m can then be replaced by $z_{me} = z_m - z_b$ in order to calculate the equivalent results for an optic used in transmission.

FORM ERROR METRICS

The two main metrics used to describe form error are:

- *Root-mean-square (RMS)* value of $z_{ra}(x,y)$, defined as:

$$\boldsymbol{error_{RMS}} = \sqrt{\frac{\mathbf{1}}{\boldsymbol{N}}\sum_{\boldsymbol{x,y}}\left(\boldsymbol{z_{ra}(x,y)}\right)^{\mathbf{2}}},$$

where N is the total number of elements in $z_{ra}(x,y)$ when x and y describe uniformly sampled points over the clear aperture,

- *Peak-to-valley (P-V)* of $z_{ra}(x,y)$, defined as:

$$\boldsymbol{error_{P-V}} = \mathbf{max}\left(\boldsymbol{z_{ra}(x,y)}\right) - \mathbf{min}\left(\boldsymbol{z_{ra}(x,y)}\right).$$

These metrics apply to the clear aperture of the optical surface only.